Let A and B be the feet of the perpendiculars from T and R to SU, respectively. Then we have

$$\frac{RT'}{TT'} = \frac{RB}{TA} = \frac{[RUS]}{[STU]} = \frac{[\bar{P}US]}{[STU]},$$

thus

$$\frac{TR}{TT'} = 1 + \frac{[\bar{P}US]}{[STU]}.$$

We finally obtain

$$\begin{aligned} &\frac{SQ}{SS'} + \frac{TR}{TT'} + \frac{UP}{UU'} \\ &= \left(1 + \frac{[\bar{P}TU]}{[STU]}\right) + \left(1 + \frac{[\bar{P}US]}{[STU]}\right) + \left(1 + \frac{[\bar{P}ST]}{[STU]}\right) = 4. \end{aligned}$$

20. Let a, b, and c be positive real numbers. Prove that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq \frac{c+a}{c+b} + \frac{a+b}{a+c} + \frac{b+c}{b+a}.$$

Solved by Arkady Alt, San Jose, CA, USA; and George Apostolopoulos, Messolonghi, Greece. We give Alt's solution.

Due to the cyclic symmetry, we can suppose that $c = \min\{a, b, c\}$.
Let x, y, and z be nonzero real numbers. Since

$$x^2z + y^2x + z^2y = 3xyz + z(x-y)^2 + y(x-z)(y-z),$$

we obtain

$$\begin{aligned} \frac{x}{y} + \frac{y}{z} + \frac{z}{x} &= \frac{x^2z + y^2x + z^2y}{xyz} \\ &= 3 + \frac{(x-y)^2}{xy} + \frac{(x-z)(y-z)}{xz}. \end{aligned}$$

Setting $x = a$, $y = b$, $z = c$ and then $x = c+a$, $y = c+b$, $z = a+b$, we obtain (respectively) the two equations

$$\begin{aligned} \frac{a}{b} + \frac{b}{c} + \frac{c}{a} &= 3 + \frac{(a-b)^2}{ab} + \frac{(a-c)(b-c)}{ac}, \\ \frac{c+a}{c+b} + \frac{a+b}{a+c} + \frac{b+c}{b+a} &= 3 + \frac{(a-b)^2}{(c+a)(c+b)} + \frac{(a-c)(b-c)}{(c+a)(a+b)}. \end{aligned}$$

Comparing the last two equations gives the result, since $(c+a)(c+b) > ab$, $(a+b)(c+a) > ac$, $(a-b)^2 \geq 0$, and $(b-c)(a-c) \geq 0$.